TOPIC NAME : DAY : TIME : DATE : / /

1.1 Propositional Logic

What is Proposition?

A Proposition is a dictarative statement that is either true(T) or false (F), but not both

True = T = 1] যদি কোনো statement কে T or F
false = F = 0] আকারে প্রকাশ করা যায় তাহলে তাকে
Proposition বলে।

Example-1: $1+1=2$

Truth value of this Proposition : T

Example-2: $2+2=5$

Truth value of this Proposition : F

Example-3: Sylhet is the Capital of Bangladesh

Truth value of this Proposition : F

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Not Proposition

> Question কখনো Proposition হতে পারে না ।
  What time is it ?

> Commands / imperative Sendense.
  Read this Carefully.

> Not Constunt / Variable
  $x+1 = 2$

> Not statement
  Bangladesh and India.

> Openion
  Emon is a best Lacturer.

> Person is not defined
  He is a college student.

TOPIC NAME : Propositional Variable DAY: TIME: DATE: / /

T এবং F বাদে যে কোনো কিছু কেই Propositional Variable হিসেবে ধরে নিতে পারবো।
যেমন: A, B, C, D, a, b, c, e, f etc.

Ex. Today is friday → p = Today is friday.

Ex. It is raining → q = It is raining

The truth value of this Propositional Variable can be true or False.

Compound Proposition

[independent clause
coordinating conjunction
independent clause

দুই বা ততোধিক Propositional variable কে যুক্ত করে যুক্ত করে যেসব Operator তাদের Connectives বলে।

Symbol	Math Name	English Name
$\neg$	Negation	Not
$\vee$	Disjunction	OR
$\wedge$	Conjunction	AND
$\oplus$	ExOR	"OR but not both
$\rightarrow$	Impleeation	"If ...... then
$\Leftrightarrow$	Equivalence / biconditional	"if & only if"

TOPIC NAME: Logical operator / Connective   DAY: TIME: DATE: / /

এটি ব্যবহার করে একটি Proposition কে পরিবর্তন করে অন্য একটি Proposition তৈরি করা হয়।

অথবা

দুই বা ততোধিক Proposition কে একত্র করতে।

(i) NOT (negation) (উল্টো যেখো)

<u>Question:</u> Find the negation of each of these proposition

(i) Today is sunday.

P = Today is sunday.

¬P = today is not sunday.

or

It is not the case that today is sunday

Truth table এর জন্য, সারির সংখ্যা = 2^n

Truth table: n = no of variable

P	¬P
F	T
T	F

input   output

TOPIC NAME : Example DAY : TIME : DATE : / /

Find the negation of each of these Proposition

(i) 6 is negative.

> 6 is not negative / It is not the case that 6 is negative

(ii) $2+1=3$

$p = 2+1=3$

$\neg p = 2+1 \neq 3$

in sentense:

"it is not the case that $2+1=3$"

(iii) "There is no pollution in new Jersey".

"There is pollution in new Jersey". or

"It is not the case that there is no pollution in new Jersey."

(iv) "The summer in marine is hot and sunny".

"It is not the case that the summer in marine is hot and sunny."

TOPIC NAME : AND
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(ii) AND / Conjunction $\wedge$ (গুন)

Today is $\underbrace{\text{Friday}}_{P}$ and $\underbrace{\text{It is raining}}_{Q}$ $\rightarrow P \wedge Q$

$\Rightarrow 2^2 = 4$ (সারি)

{ সবগুলো input সত্য হলেই কেবল Result সত্য হবে। }

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

TOPIC NAME: OR / disjunction DAY: TIME: DATE: / /

(ii) OR (V) Disjunction (যোগ)

Today is friday or it is raining

$\underbrace{\text{Today is friday}}_{P}$ or $\underbrace{\text{it is raining}}_{Q} \rightarrow P \vee Q$

P	Q	P V Q
F	F	F
F	T	T
T	F	T
T	T	T

(OR এর বেলায় যখন দুইটি input False হবে কেবল তখনই output False হবে otherwise output True হবে)

TOPIC NAME: E_xOR DAY:

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⊕ (iii) E_xOR ; XOR Exclusive OR

"student who have taken calculus or computer science can take this class".

OR

P	Q	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

" student who have taken calculus or computer science, but not both, can take the class."

XOR

P	Q	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

XOR এর ক্ষেত্রে input দুইটি যদি একই (same) হয় তাহলে Answer false হবে। otherwise True.

Ex.

যেকোনো একটি Input যদি true হয় তাহলে output true হবে।

Ex: For each of these Sentense, determine whether an exclusive OR or OR is intented

(i) Coffee or tea comes with dinner. (ExOR)

(ii) Experience with C++ or Java is required. (OR)

(iii) lunch includes soup or salad. (XOR)

(iv) you can pay using US. doller or euros. (OR) (ExOR)

or ..... but not both

TOPIC NAME : Conditional Statement DAY :
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→ Conditional Statement →

$\boxed{\text{If .... then}}$ (Implication)

$p \to q$
If p true q.

p = Today is holidays.
q = The store is closed.
→ if p then q.
$p \to q$
if Today is holiday then the store is closed.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

$p \to q$
এই অবস্থায়
T − F থাকলে False
অন্য সবায় true.

Ex. Determine whether each of these conditional statement is true or false.

F T

i) if $\underbrace{1+1=2}_{p}$, then $\underbrace{2+2=5}_{q}$

$p \to q$ → False

(ii) if $\underline{1+1}_{p} = 3$, then $\underline{2+2=4}_{q}$ → true

(iii) if 1+1=3 then dogs can fly → true

(iv) if 1+1=2 then dog can fly → False

$p \to q \Rightarrow$ p implies

6

TOPIC NAME : conditional statement DAY : TIME : DATE : / /

(i) "if p then q"

"If I am elected, then I will lower taxes".

$p \to q$

(ii) "if p, q

"If it is below freezing. It is also snowing

$p \to q$

(iii) "p is sufficient for q".

"Driving over 65 miles per hours is suffient for getting ticket".

$p \to q$

(iv) "q if p"

"Maria will get good job if she learns Discrete Mathematics

$p \to q$. $(Q \to P)$

(v) "q whenever p."

(vi) "q is necessary for p"

(vii) q follows from p"

viii) "a sufficient condition for q is p"

$p \to q$

TOPIC NAME : DAY : TIME : DATE : / /

(ix) "q when p"

"Maria will find a good Job when she learns discrete mathematics".

$p \to q$

(x) "q unless ¬p"

"Maria will find a good Job unless she does not learn discrete mathematics".

$p \to q$

(xi) "p only if"

$p \to q$

p whenever q
p if q
p when q
p unless ¬q
~~p only if q~~
p follow from q

$q \to p$

necessary for,

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE : / /

Converse, Inverse, Contrapositive

Conditional statement → new Conditional Statement
(i) converse
(ii) Inverse
(iii) contrapositive

Ex. $p \to q$

converse: $q \to p$

inverse: $\neg p \to \neg q$

Contrapositive:
i> converse
ii> inverse
$q \to p$
$\neg q \to \neg p$

Example: p = I am in sylhet.
q = I am in Bangladesh.

If I am in sylhet then I am in Bangladesh

$p \to q$

Converse: $q \to p$

If I am in Bangladesh then I am in sylhet

Inverse: $\neg p \to \neg q$

If I am not in ~~Bangladesh~~ sylhet then I am not in Bangladesh.

TOPIC NAME : DAY : TIME : DATE : / /

- contrapositive:

$q \to p$

$\neg q \to \neg p$

If I am in Bangladesh then I am not in Sylhet.

যে statement নিয়ে কাজ করবো তার সাথে contrapositive অবশ্যই match হবে।

p	q	~~p→q~~ q→p	¬p	→¬q	¬q→¬p	¬p→¬q	p→q
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
T	F	T	F	T	F	T	F
T	T	T	F	F	T	T	T
		Converse			Contrapositive	inverse	Original

Same.

TOPIC NAME : CICp (Example) DAY : TIME : DATE : / /

Quiz

> State the converse, inverse and Contrapositive of each of these following statement.

1. If it snows today, I will ski tomorrow.

p = If it snows today
q = I will ski tomorrow

$p \rightarrow q$

Converse : $q \rightarrow p$

I will ski tommorrow if it snows today.

Inverse $\neg p \rightarrow \neg q$

If it does not snows today, I will not ski tomorrow

Contrapositive: $\neg q \rightarrow \neg p$

If will not ski tomorrow, ~~then~~ then it not snows today.

If I didnot ski tomorrow then I will not have the snows today.

TOPIC NAME : DAY : TIME : DATE : / /

(ii) I come to class whenever there is going to be a quiz.

p = there is going to be a quiz.

q = I come to class.

$(p \to q)$ C

Converse: $q \to p$

I come to class ~~whenever~~ only if there is going to be a quiz.

Inverse: $\neg p \to \neg q$

There is not going to be a quiz when I come to class

Contraposition $\neg q \to \neg p$

I do not come to class only if there is not going to be a quiz.

TOPIC NAME : Biocondifional Statement DAY: TIME: DATE: / /

~~bi-implecation~~

"You can take the flight $\underbrace{\text{if and only if}}_{\text{bi-conditional}}$ you buy a ticket."

$p \leftrightarrow q$

"p is necessary and ~~only~~ sufficient for q"

"p iff q" → if and only if

"If p then q, and conversely."

Same truth value থাকলে ~~truth~~ true হবে otherwise False হবে।

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Ex. That it is below freezing is necessay and sufficient for it to be snowing.

$p \leftrightarrow q$

TOPIC NAME : Bi-conditional

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Determine whether these bioconditionals are true or False.

(i) $\underset{T}{2+2=4}$ if and only if $\underset{T}{1+1=2} \rightarrow$ T

(ii) $\underset{T}{1+1=2}$ if and only if $\underset{F}{2+3=4} \rightarrow$ F

(iii) $\underset{F}{1+1=3}$ if and only if $\underset{F}{\text{monkeys can fly}}$. T

(iv) $\underset{F}{0>1}$ if and only if $\underset{T}{2>1}$ — F

XOR & Bio-Conditional
পরস্পরের বিপরীত

TOPIC NAME : 　　　　　 DAY : [redacted] TIME : 　 DATE : / /

Truth table of compound proposition

Precedence of logical operator >

$\neg > \wedge > \vee > \rightarrow > \leftrightarrow$

negation AND OR condi bi-co

draw truth table —

(i) $(p \vee \neg q) \rightarrow (p \wedge q)$ → no of veriable = 2 (p & q)

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow p \wedge q$
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

(T-F — থাকলে False, otherwise true)

(Ex. 2) : $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$ → no of veriable = 2
সারি = $2^2 = 4$

p	q	$\neg q$	$p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
F	F	T	F	T	T
F	T	F	T	F	T
T	F	T	T	F	T
T	T	F	F	T	T

(English Sentence → Proposition)

TOPIC NAME: DAY: TIME: DATE: / /

Let p and q be the Proposition:

p = "It is below freezing".

q = "It is snowing".

Write these Propositions using p and q and logical connective

(a) It is below $\underline{\text{freezing}}_{p}$ and $\underline{\text{snowing}}_{q}$

Ans: $p \wedge q$

(b) It is $\underline{\text{below}}_{p}$ freezing but $\underline{\text{not}}_{n}$ $\underline{\text{snowing}}_{q}$".

Ans: $p \wedge \neg q$

(c) It is either $\underline{\text{snowing}}_{q}$ or $\underline{\text{freezing}}_{p}$ but not both.

Ans: $p \oplus q$

(D) If It i below freezing, it is also snowing

Ans: $p \rightarrow q$

(E) It is either below $\underline{\text{freezing}}_{p}$ or it is $\underline{\text{snowing}}_{q}$ but it is not $\underline{\text{snowing}}_{q}$ if it is below ~~snowing~~ $\underline{\text{freezing}}_{p}$

Ans: $(p \vee q) \wedge (p \rightarrow \neg q)$

13.

TOPIC NAME: DNF

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• Disjunctive Normal From

এমন একটি লজিক্যাল ফরমেট যেখানে বিভিন্ন শর্তের AND ($\wedge$) দ্বারা তৈরি ব্লকগুলো OR ($\vee$) দ্বারা যুক্ত থাকে।

Ex: $(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

TOPIC NAME: Propositional Equivalence DAY:

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$\neg(p \land q)$		$\neg p \lor \neg q$
F		F
F		F
F		F
T		T

যদি দুটি Propositional (compound) এর truth value same হয় তাহলে দুটি Proposition কে Logically Equivalence বলে।

একটির পরিবর্তে অন্যটি ব্যবহার করা যাবে।

Propositional Equivalence তিন প্রকার —

~~i> Fotu~~ ~~i> Tautog~~

(i) Tautology → Always true

(ii) Contradiction → Always False

(iii) Contingency → (True+false)

TOPIC NAME : ______ DAY : ______ TIME : ______ DATE : / /

(i) $p \vee \neg p$ (ii) $q \wedge \neg q$ (iii) $p \to q$

p	q	$\neg p$	$\neg q$	$p \vee \neg p$	$q \wedge \neg q$	$p \to q$
F	F	T	T	T	F	T
F	T	T	F	T	F	T
T	F	F	T	T	F	F
T	T	F	F	T	F	T

Tautology T | Contradiction F | mix (F & F) Contingency

Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology using truth table.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$(p \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	F	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

tautology

TOPIC NAME: Logical Equivalence. DAY: TIME: DATE: / /

কোনো Compound Proposition Logically Equivalence কিনা তা জানা / চেক করা যায় দুটি পদ্ধতির মাধ্যমে।

i) Truth table
ii) Logic Law.

Logical Equivalence using Truth table.

Example: show that $p \to q$ and $\neg p \vee q$ are logically equivalence.

p	q	$\neg p$	$p \to q$	$\neg p \vee q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

Therefore, we ~~show sho~~ say that $p \to q$ and $\neg p \vee q$ are logically equivalence.

TOPIC NAME : Logic Laws DAY : TIME : DATE : / /

(i) Double Negation law:

» $\neg(\neg p) \equiv p$

(ii) Idempotent Law:

$p \vee p \equiv p$

$p \wedge p \equiv p$

(iii) Identity Law: $p \wedge T \equiv p$

$p \vee F \equiv p$

(iv) Domination Law: $p \vee T \equiv T$

$p \wedge F \equiv F$

(v) Commutative Law: $p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

(vi) Associative Law:

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

TOPIC NAME : DAY : TIME : DATE : / /

* (vii) Inverse Law: $p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

Example: $A\bar{B}B\bar{C} + B\bar{C}C\bar{D} + \bar{A}DA\bar{B} + \bar{A}CD\bar{D}$

$A\neg B B\neg C + B\neg C C\neg D + \neg A D A\neg B + \neg A C D \neg D$

$= A.0.C + B.0.\bar{D} + 0.D.\bar{B} + \bar{A}C.0$

$= 0 + 0 + 0 + 0$

$= 0$

(viii) $p \vee (p \wedge q) \equiv p$ Absorption Law

$p \wedge (p \vee q) \equiv p$

(ix) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ Distribution Law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

* (x) Conditional Law: $p \rightarrow q \equiv \neg p \vee q$

(xi) BioConditional Law:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

TOPIC NAME: Logic Laws

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"De Morgan's Law" (1st Law)

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

n সংখ্যক চলকের জন্য এই সূত্র প্রযোজ্য হবে।

Ex. $\neg(R \wedge S \wedge T) \equiv \neg R \vee \neg S \vee \neg T$

Ex. $\neg(R \vee S \vee T) \equiv \neg R \wedge \neg S \wedge \neg T$

Exam-1: Simplify: $\neg(R \wedge S \wedge T) \wedge \neg(R \vee S \vee T)$

$$\equiv (\neg R \vee \neg S \vee \neg T) \wedge (\neg R \wedge \neg S \wedge \neg T)$$

$$= (\bar{R} + \bar{S} + \bar{T}) \cdot (\bar{R} \cdot \bar{S} \cdot \bar{T})$$

$$= \bar{R}\bar{R}\bar{S}\bar{T} + \bar{S}\bar{S}\bar{R}\bar{T} + \bar{T}\bar{T}\bar{R}\bar{S}$$

$$= \bar{R}\bar{S}\bar{T} + \bar{S}\bar{R}\bar{T} + \bar{T}\bar{R}\bar{S}$$

$$= \bar{R}\bar{S}\bar{T}$$

$$= \neg R \wedge \neg S \wedge \neg T$$

De Morgan's Law to find the negation:

Example: Use De Morgan's Law to find the negation at each at the following sentense.

(a) $\underbrace{\text{Jon is rich}}_{p}$ $\underset{\wedge}{\text{and}}$ $\underbrace{\text{happy}}_{q}$. $\rightarrow p \wedge q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Jon is not rich or not happy.

(b) Carlos $\underbrace{\text{will bicycle}}_{p}$ or $\underbrace{\text{run tommorrow}}_{q}$.

p = "Carlos will bicycle".
q = "run tommorrow".

for given sentense: $p \vee q$

Negation of $p \vee q \equiv \neg(p \vee q)$

$= \neg p \wedge \neg q$

Carlos will ~~b~~not bicycle and not run tommorrow.

$\boxed{\text{How}}$ show that each of these Conditional statement is a tautology without using truth table.

(a) $(p \land q) \to p$

$p = p \land q$
$q = p$

Now we say that, $p \to p$

now, $p \to q \equiv \neg p \lor p$

$\equiv \neg(p \land q) \lor p$

$\equiv (\neg p \lor \neg q) \lor p$ [De Morgan Law]

$\cancel{\equiv p(q \lor \neg p) \land (q \lor \neg q)}$

$\equiv \neg p \lor \neg q \lor p$

$\equiv (\neg p \lor p) \lor \neg q$

$\equiv T \lor \neg q$

$\equiv T$ (Tautology)

TOPIC NAME : DAY : TIME : DATE : / /

(b) $(p \wedge q) \rightarrow (p \vee q)$

$p' = (p \wedge q)$

$q' = (p \vee q)$

we know that from bio condition low,

$$p' \rightarrow q' \equiv \neg p' \vee q$$

so, we write

$$\equiv \neg(p \wedge q) \vee (p \vee q)$$

$$\equiv (\neg p \vee \neg q) \vee (p \vee q)$$

$$= \neg p \vee \neg q \vee p \vee q$$

$$= (\neg p \vee p) \vee (\neg q \vee q)$$

$$= T \vee T$$

$$= T \text{ (Tautology)}$$

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE : / /

Show the each of the statement are logically equivalent without using truth table.

i) $\neg(p \to q) \equiv p \wedge \neg q$

L.H.S $= \neg(p \to q)$

$= \neg(\neg p \vee q)$

$= \neg(\neg p) \wedge \neg q$

$\equiv p \wedge \neg q$

$=$ R.H.S.

(ii) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

L.H.S: $\neg(\underbrace{p}_{p} \vee \underbrace{(\neg p \wedge q)}_{q})$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ (De morgan 1st law)

$\equiv \neg p \wedge \neg(\neg p) \vee \neg q$

$\equiv \neg p \wedge (p \vee \neg q)$

$\equiv \neg p \wedge (\neg q \vee p)$

$\equiv$ ~~$\neg p \wedge p$~~

~~$\equiv (\neg p \vee \neg q) \wedge (\neg p \vee p)$~~

$\equiv$

$(\neg p \wedge \neg q) \vee (\neg p \wedge p)$

$\equiv (\underbrace{\neg p \wedge \neg q}_{p}) \vee F$

$\equiv (\neg p \wedge \neg q)$

R.H.S.

TOPIC NAME : Quantifiers

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" x can speak English",

domain = student

$P(x)$ = "x can speak English".

#কাজ: Predicate এর জন্য Value Assign করে দেওয়া।

Quantifier 2 প্রকার

i) Universal: $\forall$ (সবার জন্য)

ii) Existential: $\exists$ (কিছু অন্তত/বা এক জনের জন্য)

→ $P(x)$ = "x (student) can speak English".

~~$\forall(x)$ = "~~

$\forall x\, P(x)$ = "Every ~~body~~ student can speak English",

$\exists x\, P(x)$ = "There is a ~~stu~~ student who can speak English";

Universal

for all, for every, for each, for any, ~~for all~~ given any.

Exastantian

There exists, for some, for at least one, there is

TOPIC NAME : DAY : TIME : DATE : / /

Example: Let $Q(x)$ be the statement "$x<2$" what is the truth value of the quantification $\forall_x Q(x)$, where is the domain contain of all real number.

$Q(x) = "x<2"$

$\forall_x Q(x) =$ False

$Q(1) = 1<2$

$Q(2) = 2<2$

Example:

What is the truth value of $\forall_x P(x)$, where $P(x)$ is the statement "$x^2<10$" and the domain contain of the positive integer; not exceeding 4:

$P(x) = "x^2 < 10"$ domain $\Rightarrow x = \{0, 1, 2, 3, 4\}$

$\forall_x P(x) = "x^2 < 10"$

$\forall 4$

$=$ false

for, $\exists_x P(x) = "x^2 > 10"$

true

5

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TOPIC NAME : 13 Predicates & Quantifiers

DAY :

TIME : DATE : / /

i) " x is greater than 3" → $P(x)$ — U

ii) " x is greater than y". → $P(x,y)$ — B

iii) " x is greater than y and z" → $P(x,y,z)$ — T

iv) " x can speak English ". → $P(x)$ — U

x — veriable or subject

can speak English — Predicate.

$P(x) \rightarrow x$ is a Predicate of P

$P(x)$ = x is greater then 3.

$x = 2$, $P(2)$ = 2 is greater then 3 (Proposition)

Type of Predicates

i) Unary : 1 veriable

ii) Binary : 2 veriable.

iii) Ternary : 3 veriable.

TOPIC NAME : DAY : TIME : DATE : / /

Ex. Let $P(x)$ denote the statement "$x > 3$", what are the truth value of $P(4)$ and $P(2)$?

$P(x) =$ "$x > 3$".

$P(4) =$ "$4 > 3$"

∴ Truth value : True

$P(2) =$ "$2 > 3$"

∴ Truth value : False.

Ex. Let $Q(x, y)$ denote the statement "$x = y + 3$". What are the truth value of the propositions $Q(1,2)$ and $Q(3,0)$.

$Q(x, y) =$ "$x = y + 3$"

for, $Q(1,2) =$ "$1 = 2+3$" truth value = false.

for $Q(3,0) =$ "$3 = 0+3$" Truth value = ~~false~~ true.

TOPIC NAME : ~~Negating Quantified Expression~~ TIME : DATE : / /

Logical expression to English

$C(x)$ is "x is a comedian".
$F(x)$ is "x is funny".
domain → all people.

a) $\forall x\,(C(x) \rightarrow F(x))$

For all x, if x is a comedian then x is a funny.

→ every commedian is funny.

(b) $\forall x\,(C(x) \land F(x))$

for all x, x is a comedian and x is funny.

↳ Every person is a funny commedian.

TOPIC NAME: Negating Quantified expression. TIME: DATE: / /

$\forall_x P(x)$

$\equiv \neg(\forall_x P(x))$

$\equiv \exists_x \neg P(x)$

$\exists_x P(x)$

$\equiv \neg(\exists_x P(x))$

$\equiv \forall_x \neg P(x)$

Ex. "Every student in your class has $\underbrace{\text{taken a course in calculus}}_{C(x)}$".

$\forall_x C(x)$

$= \neg(\forall_x C(x))$

$= \exists_x \neg C(x)$

= There is a student in your Class who have not taken a courses in ~~elec~~ calculus.

Ex. "There is an $\underbrace{\text{honest politician}}_{C(x)}$",

$\exists_x C(x)$

$\neg(\exists_x C(x))$

$\Rightarrow \forall_x \neg C(x)$

$\Rightarrow$ Every politician did not honest.

np TOPIC NAME: Nested Quantifiers DAY: TIME: DATE: / /

Translate the following statement into English.

$\forall x \forall y\,(x>0) \wedge (y<0) \rightarrow (xy<0)$

Domain: real numbers.

For every real number, x and y,

if x is possive
and y is negative
then xy is negative.

Translate the following statement into a logical Expression.

"The sum of two positive numbers/integer is always positive",

Ans. $(x>0) \wedge (y>0) \rightarrow (x+y>0)$

$\forall x\,(C(x) \vee \exists y\,(C(y) \wedge F(x,y)))$

$C(x)$ = "x has a computer".

$F(x,y)$ = "x and y are friend".

Domain → All students

For Every student x, in your school, x has a Computer on there is a student y has a computer and

TOPIC NAME : Rules of Inferance → (অনুমান)

DAY:

TIME: DATE: / /

Valid Argument in Propositional logic:

" If you have a current password, then you can log on to the network. $(p \to q)$ | premise

p

Therefore you can log on to the network. $\therefore q$ → conclution

Valid = Primese $\boxed{(T)}$ + conclusion $\boxed{(T)}$

Rules of Interfance

① Modus Ponens:

Ex. If it is raining, then I will study discreate mathematics.".

" If it is raining". Therefore, I will study discreate mathematics.

$p \to q$

p

---

q

TOPIC NAME : DAY :
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(ii) Modus Tollens:

Ex. "If it is ~~snowing~~ raining, then I will study math"
"I will not study discrete math"

$$p \rightarrow q$$
$$\neg q$$
$$\therefore \neg p$$

(3) Hypothetical Syllogism.

Ex. "If it is raining, then I will study discrete math".
" If, I study discrete math, I will get an A".

$p \rightarrow q$ | $p \rightarrow q \rightarrow r$ (3) $(p \rightarrow r)$

~~$p \rightarrow q$~~
$q \rightarrow r$

$\therefore p \rightarrow r$

TOPIC NAME : DAY : TIME : DATE : / /

④ Disjunctive Syllogism:

Ex. "I will study discrete math or I will study English Literature". (p = I will study discrete math, q = I will study English Literature)

" I will not study discrete math".

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

⑤ Addition:

$$\begin{array}{l} p \longrightarrow T \\ \hline \therefore p \vee q \longrightarrow T \end{array}$$

$$\begin{array}{l} p \longrightarrow T \\ \hline \therefore p \vee q \longrightarrow T \end{array}$$

p= "I will DM".
q = " " study General math".

$$\begin{array}{l} \therefore p \\ \hline \therefore p \vee q \end{array}$$

TOPIC NAME: DAY: TIME: DATE: / /

(6) Simplification:

$$\frac{\overset{T}{p} \wedge \overset{T}{q} = T}{\therefore p}$$

p = "I will study English",
q = "I will study Math".

$$\frac{p \wedge q}{\therefore p}$$
or. $\therefore q$

/ কারন উভয় সত্য | যেকোনো একটি।

(~~Add~~ and কে remove করা হবে।)

(7) Conjunction:

$$\frac{\begin{array}{l} p \rightarrow T \\ q \rightarrow T \end{array}}{p \wedge q - T}$$

~~Add~~ ~~Add~~
$\boxed{\text{and Add করা হবে}}$

p = "I will study English".
q = "I will study Math",
"I will study English and I will study Math";

TOPIC NAME : DAY : TIME : DATE : / /

(8) Resolution:

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

p = "I will study English".
q = "I will study Math".
r = "I will study Bangla".

Ex-1: From the single Proposition:

$p \wedge (p \to q)$

show that q is a conclusion:

Sol: $p \wedge (p \to q) \longrightarrow$ Primise

$= p$ [use simplification]

$= (p \to q)$ [use simplification in line 1]

$= q$ [line 2 & 3 use Modus ~~Pollens~~ ponens]

TOPIC NAME : DAY : TIME : DATE : / /

Ex.-2 1. $T \to (M \vee E)$
2. $S \to \neg E$
3. $T \wedge S$

show that M is a conclusion

Sol:
1. $T \to (M \vee E)$
2. $S \to \neg E$
3. $T \wedge S$
4. T [simplefication line 3]
5. S [simplefication line 3]
6. $M \vee E$ [use Modus ponent in line 4 & 1]
7. $\neg E$ [use Modus ponent in line 5 & 2]
8. M [6, 7 line use disjunctive syllogism]

TOPIC NAME :   DAY :
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Ex.-3:

show that $\neg p$ is a conclusion

1. p
2. $p \to q$
3. $q \to \neg p$

Sol: 1. p
2. $p \to q$
3. $q \to \neg p$
} premise

4. q [use Modus ponent in line 1 & 2]
5. $\neg p$ [use Modus ponent in line 3 & 4]

Ex. show ~~that~~ the hypothesis.

1. "It is not sunny this afternoon and it is colder than yeasterday.

p = " It is sunny this afternoon".

q = It is colder than yeasterday".

~~q = "yeasterday".~~

$\neg p \wedge q$

TOPIC NAME: Example-1: Show the hypothesis

DAY:
TIME: DATE: / /

1. " It is not sunny this afternoon and it is colder than yeasterday".

2. " We will go swimming only if it is sunny".

3. If we didnot go swimming then we will take a canoes trip".

4. If we take a canoe trip, then will be home by sunset ".

Lead to the conclusion "we will be home by sunset.

p = " It is sunny this afternoon".

q = " It is colder than yeasterday".

r = " we ~~go~~ will go swimming

s = " we will ~~at~~ take a canoes trip"

t = " will be home by sunset

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$
5. $\neg p$ [line 1, simpletication]
6. $\neg r$ [Modus tollens line- 2 & 5]
7. s [3 ,6 lime- use Modus ponest]
8. t [line 4 &, 7 use Modus ponens]

TOPIC NAME: Introduction to Sets DAY: TIME: DATE: / /

Methods to Represent sets:

(i) Roster Method:

Example: set of vowels; $V = \{a, e, i, o, u\}$

(ii) set builder notation:

Ex. The set of O of all odd positive integers less than 10 can be written as

$O = \{x : x$ is an odd positive integer less than $10\}$

Roster form: $O = \{1, 3, 5, 7, 9\}$

Example of sets:

Standard set:

i) Natural number: $N = \{0, 1, 2, 3, 4, 5 \dots\}$

ii) Integers: $Z = \{\dots -2, -1, 0, 1, 2 \dots\}$

iii) positive integers, $Z^+ = \{1, 2, 3, 4, 5 \dots\}$

TOPIC NAME : DAY : TIME : DATE : / /

(iv) Real number: $R = \{47.3, -12, 3, \pi, \ldots\}$

(v) Rational number, $Q = \{1.5, 2.6, -3.8, 8, 15 \ldots\}$
(ভগ্নাংশ/মূলদ)

Set Equality:

দুটি set এর সকল element same এবং element সংখ্যা সমান হলে সেট দুটিকে Equal সেট/set বলা হয়।

$A = \{1, 2, 3\}$, $B = \{2, 3, 1\}$

$\Rightarrow$ A&B set are equal

Ex-2: $A = \{9, 2, 7, -3\}$, $B = \{7, 9, -3, 2\}$

$A = B$

(iii) $A = \{$dog, cat, horse$\}$ $B = \{$dog, cat, horse, Rabit$\}$

$A \neq B$

Order / Repeatation Matter করে না।

TOPIC NAME: Multiply

DAY:

TIME: DATE: / /

Cartesian Product of sets:

Cartesian Product: $A \times B$

$B \times A$

[A আগে থাকলে A দিয়ে / B আগে থাকলে B দিয়ে]

Ex-i: $A = \{good, bad\}$ $B = \{Student, sir\}$

$A \times B = \{(good, student), (good, sir), (bad, student), (bad, sir)\}$

ϕ

$(B \times A) = \{(studnt, goo), (student, bad), (sir, good), (sir, bad)\}$

TOPIC NAME : DAY : TIME : DATE : / /

Power set : P(A)

Cardinality : element সংখ্যা।

$S = \{a, b, c\}$

$|S| = 3$

$|\phi| = 0 \rightarrow$ empty set

P(A) = "power set of A"
Also written as 2^A

$2^n =$

$A = \{x, y, z\}$ এর power set হবে লিখে।

Ex. $P(A) = \{\{\phi\}, \{x\}, \{y\}, \{z\}, \{x,y\}, \{x,z\}, \{y,z\}, \{x,y,z\}\}$. (check $= 2^3 = 8$)

Ex. What is a power set of the empty set.

$\Rightarrow P(\{\phi\}) = \{\phi, \{\phi\}\}$

Sub-set :

$A \subseteq B$

Sub-set : $A \subseteq B$ [A এর সব উপাদান B তে আছে]

$B \subseteq A$ [B " " " A এ আছে]

Set - Operations :

① Union : $\cup$: $A \cup B = \{x : x \in A \vee x \in B\}$

Ex. $A = \{a, b\}$, $B = \{b, c, d\}$

$A \cup B = \{a, b, c, d\}$

(ii) Intersection : $\cap$: $A \cap B = \{x : x \in A \wedge x \in B\}$

Ex. $A = \{a, b\}$, $B = \{b, c, d\}$

$A \cap B = \{b\}$

Disjoint : two set → $\cap$ → empty

$$A \cap B = \emptyset$$

Ex. $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$A \cap B = \emptyset$ → Disjoint

Difference : $A - B = \{x \mid x \in A \cap x \notin B\}$

Ex. $A = \{a, b\}$, $B = \{b, c, d\}$

$A - B = \{a\}$

$B - A = \{c, d\}$

Complement : $A^c / \bar{A} / A' / A^{\circ}$

(পূরক)

Universal set দেওয়া থাকলে : $A^c = U - A$

$U = \{3, 5, 7, 8, 9, 10, 12\}$ $A = \{8, 9, 10, 12\}$

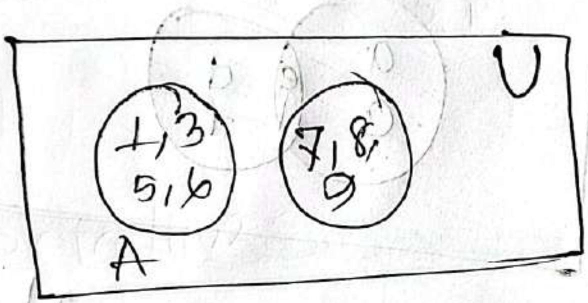
$A^c = U - A$
$= \{3, 5, 7\}$

TOPIC NAME: ~~XXX~~ DAY:
TIME: DATE: / /

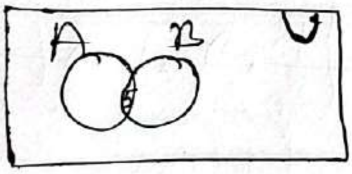
Ven diagram

$A = \{1, 3, 5, 6\}$, $B = \{7, 8, 9\}$

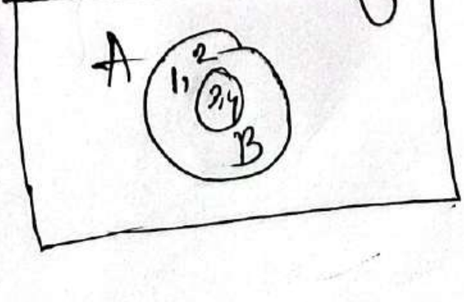
Disjoint:



Overlapping: $A = \{1, 3, 5\}$, $B = \{5, 6, 8\}$



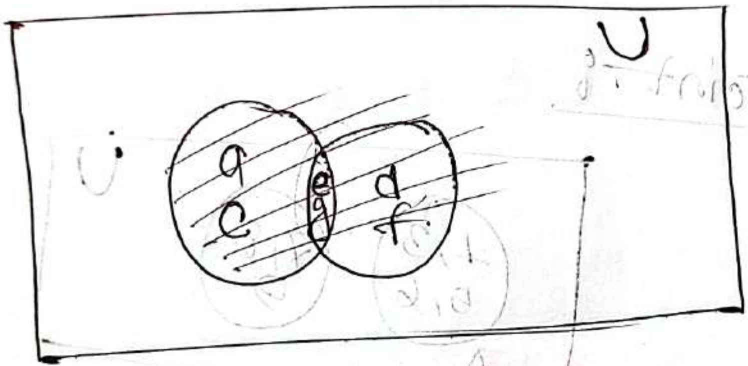
Subset: $A = \{1, 3\}$, $B = \{1, 3, 7, 8\}$



TOPIC NAME : DAY : TIME : DATE : / /

Union:

$U = \{a, b, c, d, e, f, g,\}$, $A = \{a, c, e, g\}$, $B = \{d, e, f, g\}$



$A \cup B = \{a, c, e, f, g, d\}$

$A \cap B = \{e, g\}$

TOPIC NAME :

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Ex. Given that A and B are two mutually exclusive set such that,

$A - B = \{1, 2\}$

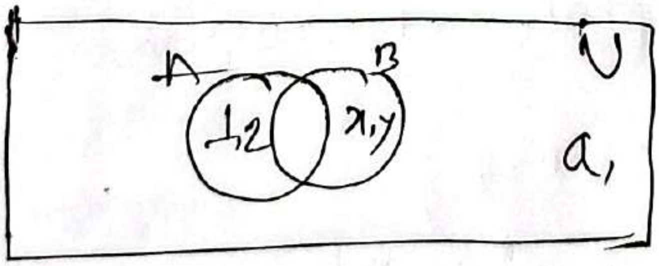
$A \cup B = \{1, 2, x, y\}$

universal set $= \{1, 2, x, y, a\}$

(i) Repreasent the information above in the venn diagram.

(ii) Find $P(B \times (A \cup B)')$

Solution



TOPIC NAME : DAY : TIME : DATE : / /

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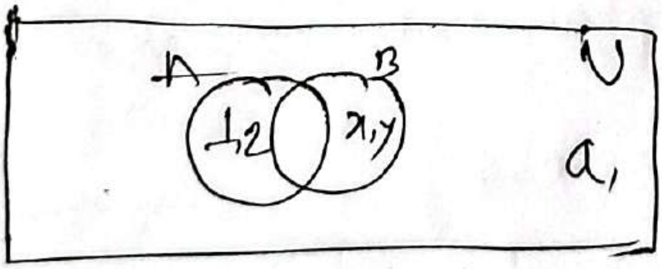

TOPIC NAME :
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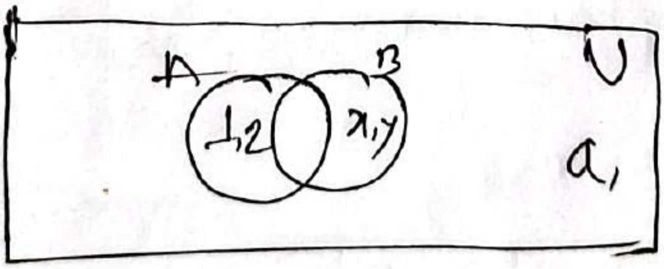


TOPIC NAME : # Function #   DAY :
TIME :   DATE :   /   /

function

if f is a function from A to B, we write,

(i) $f : A \to B$ (Here " $\to$ " has nothing to do with if .... then)

Domain   Co-Domain

(ii) $f(a') = b \to$ b is image
a = pre-image

Domain, ~~cons~~ function এ যে value দেওয়া হয়,

Co-Domain: possible come-out of a function

Range: actually comes out of a function.

Ex. (i) $x \to 2x+1$

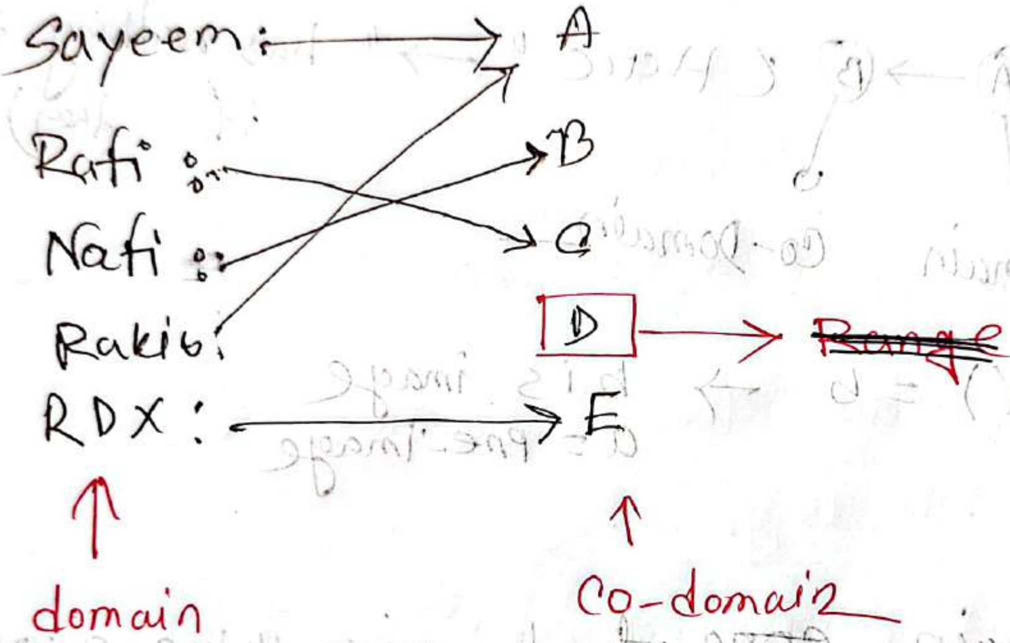
Domain: $\{1, 2, 3, 4, 5\}$

Co-domain = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Range = $\{1, 3, 5, 7, 9\}$

TOPIC NAME: DAY: TIME: DATE: / /

Q: What are the domain, Co-domain and range of the function that assigns grades to students



d: {Sayeem, Rafi, Nafi, Rakib, RDX}.

Co-d: {A, B, C, D, E}

Range {A, B, C, E}

TOPIC NAME :

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Sum and Product of function :

f_1, & f_2 two function -

(i) sum: $(f_1 + f_2)(x) = f_1(x) + f_2(x)$

(ii) Product $(f_1 f_2)(x) = f_1(x) \times f_2(x)$.

Ex. $f_1(x) = 3x$, $f_2(x) = x+5$

sum: $(f_1 + f_2)x = f_1(x) + f_2(x) = 3x + x + 5$

$= 4x + 5$

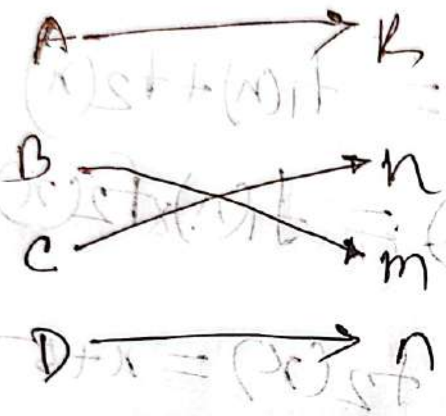
product: $(f_1 f_2)x = f_1(x) \times f_2(x) = 3x \cdot (x+5)$

$= 3x^2 + 15x$

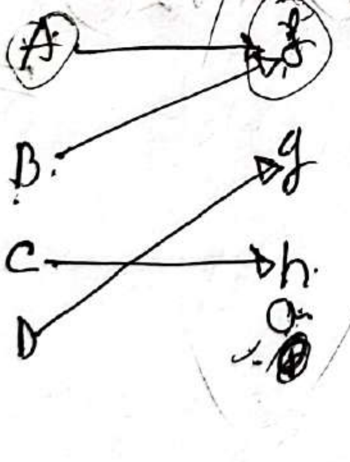
GOOD LUCK™

#propertice of function

① one to one function / $\boxed{\text{Injaective:}}$



onto function / surjuctive



প্রত্যেকটি element
ব্যবহার হতে হবে।

TOPIC NAME :
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Bi-Juctive :

one to one corresponding

দুটি প্রকারের
injective + surjective

inverse of function :

Inverse হওয়ার জন্য function কে অবশ্যই Bijective হতে হবে।

TOPIC NAME : DAY : TIME : DATE : / /

floor : $\lfloor x \rfloor = \lfloor 2.3 \rfloor = 2$

ceiling : $\lceil x \rceil = \lceil 2.3 \rceil = 3$

Ex. (i) $f: N \to N$, $f(x) = x^2$; (i) Is f one to one
(ii) is f onto ?

$f(x) = x^2$

$f(x_1) = x_1^2$

$f(x_2) = x_2^2$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$ $\Rightarrow$ It is not one to one

$x_1 = \pm\sqrt{x_2^2}$

$x_1 = +x_2$

$x_1 = -x_2$

TOPIC NAME : DAY : TIME : DATE : / /

(ii) Let $f(x) = y$

$\Rightarrow x^2 = y$

$x = \pm\sqrt{y}$

y is real number, so it can be negative too

There ~~is no~~ x^2 is not onto.

(ii) Find the inverse of $g(x) = \frac{x-3}{x-2}$

$$g(x) = \frac{x+3}{x-2}$$

Let $g(x) = y$, $y = \frac{x+3}{x-2}$

$$y(x-2) = x+3$$

$$\Rightarrow xy - 2y = x+3$$

$$\Rightarrow xy - 2y - x = 3$$

$$\Rightarrow x(y-1) - 2y = 3$$

$$\Rightarrow x = \frac{3+2y}{y-1}$$

$$g^{-1}(y) = \frac{2y+3}{y-1}$$

$$g^{-1}(x) = \frac{2x+3}{x-1}$$